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# A FEW REMARKS ON THE ENERGY TRANSFER ANALYSES DURING A TIME LESS THAN PERIOD T FOR VOLTAGE AND CURRENT WAVEFORMS IN ELECTRICAL CIRCUITS

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**Abstract:** According to the definition the effective value of voltage $V$ is equal to $V_{rms}$ as to the mean value of $u(t)$ during the energy transfer time $T$ or $kT$, where $k$ may be 1, 2, 3, 4 etc. However, in power electronics circuits (e.g. rectifiers, choppers, converters, inverters) the energy transfer time, $\Delta t$, in many situations is less than the functions period, $T$ ($\Delta t < T$). In such a situation the author asks a question – how can we calculate the effective value of voltage $V$?

**Keywords:** energy transfer time, effective voltage, effective current, Fryze's concept.

## 1. INTRODUCTION

According to Fryze [Fryze 1933] the effective current value $V$ is based on the following physical observation: "Joule's heat does not depend on the direction of the current; this occurs with direct current (DC) as well as alternating current (AC)". Based on this, all definitions stated that for any periodical voltage $u(t)$ and for any periodic current $i(t)$ the effective voltage $V$ and effective current $I$ are equal to $V_{rms}$ or $I_{rms}$. This means that the RMS voltage/current value can also be defined as the value of the direct voltage/current that dissipates the same power in a resistor. In normal engineering calculations, both values are calculated separately as $V=V_{rms}$ and $I=I_{rms}$ independent of the situation, whether there is or not energy transfer in the circuit. In relation to this situation, the author raises the question: do we always calculate the effective value $V$ or $V_{rms}$ in electrical circuits properly?

## 2. SOME HISTORICAL REMARKS

### 2.1. Effective voltage (V) and current (I) calculations

The dissipation of Joule's heat in circuits with direct current *I* can be written in the form:

$$Q_{DC} = cRI^2\Delta t \tag{1}$$

and in circuits with alternating current $i(t)$ it takes the form:

$$Q_{AC} = c\int_{t_1}^{t_2} Ri^2(t)\,dt \tag{2}$$

where:

$Q$ – heat energy,
$c$ – constants,
$\Delta t = t_2 - t_1$ is the energy transfer time.

If $\Delta t = T$ or ($\Delta t = T + kT$ where $k = 1, 2, 3\ldots$), and $R$ is (LTI – *Linear Time Invariant*) during the time when the electric energy is converted into energy heat (this means that during the time when there is energy transfer form source to load), the following equation has been stated $Q_{DC} = Q_{AC}$ during introduction of „the effective value *I*" of periodical current $i(t)$, so:

$$cRI^2T = cR\int_0^T i^2(t)\,dt \tag{3}$$

After manipulation into equation (3) one can achieve:

$$I^2 = \frac{1}{T}\int_0^T i^2(t)\,dt$$

or

$$I = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt} = I_{rms} \tag{4}$$

Equation (4) states that the effective value *I* of any periodic current $i(t)$ is equal to the root mean square (*rms*) value of the $i^2(t)$ current. The right part of equation (4) is a mathematical abstract known as "a norm" of function $i(t)$ and it is written as a symbol, $\|i\|$. However, in electrical engineering, the effective values *I* or *V* of any periodic current $i(t)$ and voltage $u(t)$ are not a "*mathematical abstract*" due to the fact that both values pose preciously defined physical interpretation.

The current effective value (4) has the following interpretation:

The **effective value *I* of any periodic current *i*(*t*)** having a period *T*, **is a value of direct current *I***, which converts the electrical energy into heat energy on resistor *R*(LTI), as current *i*(*t*) does during the same time equal to *T* or equal to *kT* where *k* = 1, 2, 3…

The Standard [IEEE Std 1459, 2010] states that the effective value *I* of any periodic current *i*(*t*) as $I_{RMS} = I$ only, without any additional comments. The author has found in the Standards and other formal publications no information as to what must be fulfilled such that *I* is equal to $I_{RMS}$. In the author's opinion, two conditions must be drawn from equations (3, 4, 5). The conditions where:

$$I = I_{rms} \tag{5}$$

These are as follows:

5a) *R* must be an LTI (Linear Time Invariant) during the transfer energy time from electrical source to load.

5b) The energy transfer time, $\Delta t$, must be equal to the period *T* or a multiplication of this time, $\Delta t = kT$, where: $k = 1, 2, 3$, and so on.

If condition (5b) is not fulfilled, let's say because $\Delta t < T$; $\Delta t = t_2 - t_1$, the effective value *I* of current must be calculated in period $\Delta t$:

$$I = \sqrt{\frac{1}{T}\int_{t1}^{t2} i^2(t)\, dt} \tag{6}$$

Because current *i*(*t*) is the necessary factor during the transfer of energy from the electrical source to load *R* (LTI), the effective value *I* is always equal to $I_{RMS,}$ so one can write $I = I_{rms}$. However, this identity is not always related to voltage *u*(*t*). Based on Ohm's law for an instantaneous value of voltage and current in DC and AC circuits, one can write:

$$u\,(t) = Ri(t);\; i(t) = \frac{u(t)}{R} \tag{7}$$

Using equation (5b) one can calculate the effective voltage value, *V*, as follows:

a) For *R* (LTI) and $\Delta t = T$, equation (3) takes the form:

$$cR\frac{U^2}{R^2}T = cR\int_0^T \frac{1}{R^2}u^2(t)\, dt \tag{8}$$

and after mathematical manipulation:

$$V^2T = \int_0^T u^2(t)\, dt \tag{9}$$

$$V^2 = \frac{1}{T}\int_0^T u^2(t)\,dt \tag{10}$$

and finally

$$V = \sqrt{\frac{1}{T}\int_0^T u^2(t)\,dt} = V_{RMS} \tag{11}$$

b) For $R$ (LTI) but if $\Delta t = (t_2 - t_1) < T$, equation (11) takes the form:

$$V = \sqrt{\frac{1}{T}\int_{t_1}^{t_2} u^2(t)\,dt} < V_{RMS} \tag{12}$$

It is not difficult to observe that $V < V_{RMS}$. In this case the proof is as follow: in both cases the equations 11a) and 11b) (3) must be fulfilled while the energy equality,
$E_{DC} = P_{DC} \cdot \Delta t = E_{AC}$, implies power equality:

$$P_{DC} = P_{AC} \tag{13}$$

so:

$$\underbrace{IV}_{DC} = \underbrace{IV}_{AC} \tag{14}$$

and as a consequence:

$$V = \frac{P}{I} = \frac{P}{I_{rms}} \tag{15}$$

One can observe that the effective value, $V$, of the periodic voltage, $u(t)$, is not always equal to $V_{rms}$. To fulfil equality, $V = V_{RMS}$, condition (6) must be satisfied. It was Fryze who indicated that the energy transfer time must be the same during the calculations for $I$ and $V$.

### 2.2. Calculating apparent power S

Apparent power $S$ in circuits with a periodic sinewave voltage and current is defined as follow [IEEE Std 1459, 2010]:

$$S = V_{rms} I_{rms} \tag{16}$$

Eq. (16) assumes that condition (6) is fulfilled so that $V = V_{rms}$ and $I = I_{rms}$. In electrical circuits in which energy transfer from the source to the load does not fulfil equation (6), the apparent power $S$ should be calculated in the form:

$$S = VI = VI_{RMS} \tag{17}$$

However, such an equation as (17) does not exist in the standard [IEEE Std 1459, 2010].

A very interesting apparent power definition was given by Lyon [Lyon 1935] and Filipski [Filipski 1993]. According to them, in circuits with a sinewave voltage and current, the apparent power $S$ is:

a) "For single-phase systems, operating under sinusoidal conditions, **the apparent power of a load** or a cluster of loads supplied by a feeder **is the maximum active power** that can be transmitted through the feeder, while keeping the receiving end rms voltage and the feeder\variable losses constant.

This definition can also be extended to a source: **The apparent power of a source is the maximum active power** that can be supplied, or generated by, the source, while keeping its output voltage and the internal variable power losses constant" [Lyon 1935].

The above definition was introduced in a modified form by W.V. Lyon in 1920, promoted by A. Lienard in 1926, and later advocated by H.L. Curtis and F.B. Silsbee.

b) "**Apparent power is numerically equal to the maximum active power** that exists at given points of entry for a given effective value of the sinusoidal current and the potential difference, and hence is directly related to the size of the required equipment and to the generation and transmission losses" [Filipski 1993].

It must be underlined that, if condition (6) is not fulfilled, there is no apparent power $S$ definition and there is no sense in using definition (16).

### 2.3. Budeanu'S postulateS

In his paper [Budeanu 1927], Budeanu postulated introducing two aspect of optimal energy transfer in electrical circuits. In his opinion, the optimal energy transfer takes place in the circuit if the periodic functions of voltage and current are proportional during the energy transfer time. This means that load is resistive $R$ and has LTI features so that $u(t) = R\, i(t)$. These conditions require that during the energy transfer time:

a) both voltage/current functions have the same shape e.g. sine waveform;
b) there is no phase shift between voltage and current (if shape is sinewave) or there is no delayed time between voltage and current.

It is worth pointing out Budeanu's postulate have a universal character related to any shape of periodic voltage and current functions. If the voltage and current have the same sine waveforms with the phase shift φ, the active power $P$ is equal to the well-known formula $P = I_{rms}V_{rms}\cos\varphi$. This formula can be written as $P = VI$, where:

$$\boldsymbol{I = I_{rms}} \text{ and } \boldsymbol{V = V_{rms}\cos\varphi}.$$

## 3. A CIRCUIT WITH TWO DIODE CALCULATIONS

Let us calculate the $V_{rms}$, $I_{rms}$ and $V$ in the circuit from Fig.1 if $u(t) = V\sqrt{2}\sin\omega t$.

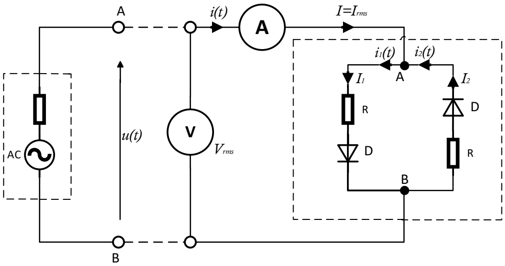


**Fig. 2.** The circuit with the R and D load: where: $V = V_{rms}$ the effective value of the source voltage, known as the secondary voltage, D – ideal diode; R (LTI)

The following voltage $u(t)$, current $i(t)$ and power $p(t)$ waveforms are presented in Figure 2.

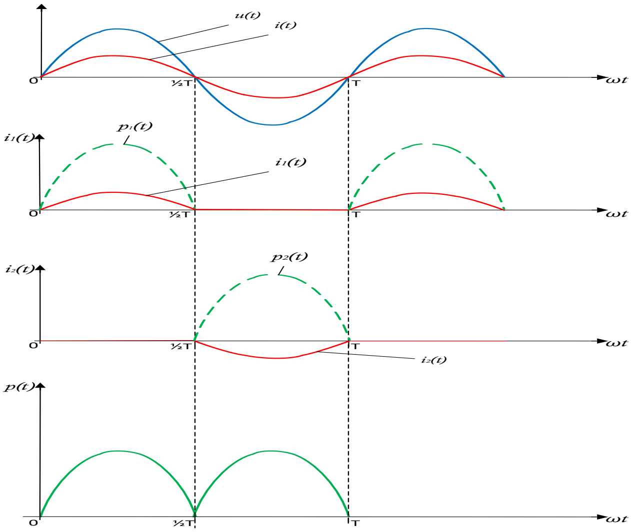


**Fig. 2.** Illustration of the instantaneous voltage, current and power waveforms in the circuit from Figure 1

The current and voltage calculations are as follows:

a)

$$I = I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\ \mathrm{d}t} \tag{18}$$

$$V = V_{rms} = \sqrt{\frac{1}{T}\int_0^T u^2(t)}\ \mathrm{d}t \tag{19}$$

b)

$$I_1 = \sqrt{\frac{1}{T}\int_0^{T/2} i^2(t)}\ \mathrm{d}t = \frac{I}{\sqrt{2}}$$

$$I_2 = \sqrt{\frac{1}{T}\int_{T/2}^{T} i^2(t)}\ \mathrm{d}t = \frac{I}{\sqrt{2}} \tag{20}$$

so that:

$$I_1 = I_2 \tag{21}$$

The active power *P:*

$$P = \frac{1}{T}\int_0^T u(t)\cdot i(t)\ \mathrm{d}t = V_{RMS}\cdot I_{RMS} = V\cdot I \tag{22}$$

Equation (22) can be written in a different form:

$$P = \frac{1}{T}\int_0^T p(t)\,\mathrm{d}t = \frac{1}{T}\left(\int_0^{T/2} p(t)\ \mathrm{d}t + \int_{T/2}^{T} p(t)\,\mathrm{d}t\right) = P_1 + P_2$$

and $P_1 = P_2$.

Question: What are the values of $V_1$ or $V_2$?

If $P_1 = P_2 = \frac{P}{2}$ and $I_1 = I_2$ so:

$$P_1 = V_1 I_1 \tag{23}$$

and:

$$P_2 = V_2 I_2 \tag{24}$$

Based on equations (20), (21) or (22), one can obtain:

$$\frac{P}{2} = V_1\frac{I}{\sqrt{2}} = \frac{VI}{2} \tag{25}$$

From equation (25):

$$V_1 = \frac{V\sqrt{2}}{2} = \frac{V}{\sqrt{2}} < V \tag{26}$$

To check equation (26) one can conclude that if $V_1 = V_2 = \frac{V}{\sqrt{2}}$ so:

$$P = \underbrace{P_1}_{V_1 I_1} + \underbrace{P_2}_{V_2 I_2} = \left(\frac{V}{\sqrt{2}}\cdot\frac{I}{\sqrt{2}}\right) + \left(\frac{V}{\sqrt{2}}\cdot\frac{I}{\sqrt{2}}\right) = \frac{P}{2} + \frac{P}{2} = P \tag{27}$$

and the left side of equation (27) is equal to right side.

The calculation confirms that even $R$ is LTI during the energy transfer time $\Delta t < T$ the effective voltage $V$ is not equal $V_{rms}$ ($V \neq V_{rms}$).

## 4. CONCLUSIONS

Based on the analysis and calculations presented in p.2 and 3, one can draw the following conclusions:

1. Energy transfer from the source to the load in circuits with periodic alternating waveforms are based on the assumption of equality of the active power $P_{DC} = P_{AC}$ during the same energy transfer time, $\Delta t$.
2. Voltage/current definitions of $V_{RMS}$, $I_{RMS}$ should be completed by condition (6).
3. Equity $I = I_{rms}$ is always true, but $V = V_{rms}$ is not always. If condition (6) is not fulfilled the best way is to calculate effective voltage $V$ based on the active power, $P$:

$$V = \frac{P}{I} < V_{rms}\,.$$

In the author's opinion these considerations are in line with Fryze's concept based on the "voltage components" (not on the well-known "current components"). This means that the current is the "master" and the voltage is the "slave" in any circuits independent of voltage/current waveforms.

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